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RESEARCH ON

SOLID PROPELLANT COMBUSTION INSTABILITY

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ABSTRACT

Progress made during the second quarter of a research program directed toward a fundamental study of the non-steady combustion of solid propellants with application to rocket instability is described.

A film strip of a metallized solid propellant burning in an oscillating pressure field was analyzed in order to determine the phase relationship between the imposed pressure maxima and an apparent wave of luminosity in the combustion gases. The implications of these observations as they affect the model of the burning propellant are briefly considered.

Two types of experimental observation which will be used to study non-steady combustion are outlined. One method involves the measurement of the temperature of the combustion gases downstream from the flame zone. The use of a "particle track" method for the observation of pressure-velocity relationships in the burnt gases is also considered.

I. INTRODUCTION

The complex interaction of an oscillating pressure field with the flame of a burning solid propellant, which apparently lies at the heart of the non-steady burning problem, is given further consideration in this second quarterly progress report. Earlier work, in which the feasibility and efficacy of a number of frequently discussed experimental techniques were investigated, highlighted the difficulty of contriving experiments capable of yielding sufficiently sensitive measurements of the acoustic response of a burning surface. In this report, experiments are described which seem capable of yielding results pertinent to the acoustic response problem. At the same time, experimental data obtained in a separate investigation have been reduced in a new way to give information regarding the phase relationship between a pressure oscillation and a response of the combustion process.

II. FILM ANALYSIS

The burning of metals incorporated in a solid propellant matrix has been under investigation by Dr. William A. Wood of Rohm & Haas Co. (Redstone Division) for some time. As part of this study, cinematography was employed to record the burning process of a metal-containing propellant under conditions of oscillating chamber pressure. The experimental setup is as shown in Figure 1. It was noted that, in the combustion gases from a propellant containing 2% magnesium, there were clearly visible luminous zones. Figure 2 is a sketch of a typical frame, showing the dark and luminous zones. These zones moved with the product gases, but the separation between them oscillated with time at the same frequency as the pressure oscillations. The experimental conditions for the particular film strip studied are outlined in Table 1.

The phase difference between the luminous waves and the pressure cycle is of fundamental importance to an understanding of the combustion mechanism. Unfortunately, no pressure-time history was available with the film strip. However, it was possible to obtain the pressure-time history in the cavity from the variation in the spacing of the luminous zones. These variations in spacing arise from oscillations in the gas density which are in turn related to the pressure oscillations in the gas column. An analysis was made of about 500 frames, from which was found an average phase lag of 130° between a pressure maximum and an emission of a luminous zone from the burning surface.

In general a phase lag might be caused by at least two conditions. First, an oscillatory variation in the propellant combustion process might result in the emission of a type of "smoke" periodically. Of course, periodic increases in pressure cause corresponding increases in gas evolution rate at the surface, but it is the variations in <u>mass fraction</u> of smoke that would be carried with the stream and show up in the photograph. In this case, the MgO particles could become visible by acting as reflectors of the light from the burning propellant surface. The phase relation here can only be conjectured.

Second, the luminous zones could represent temperature peaks resulting from entropy waves. These entropy waves are carried with the stream and are caused by the fact that the flame temperature holds constant while the pressure oscillates. If so, the phase lag is expected to be 180°. The finding of a phase lag of 130° is obviously a matter to be investigated in further experiments. These experiments would distinguish between thermal luminosity or "smoke" as the fundamental cause for the visible bands.

III. THE ACOUSTIC ADMITTANCE

McClure and coworkers (1) have suggested that the acoustic admittance "Y" of the burning surface is an important parameter in determing the stability of a rocket chamber. The equation for acoustic admittance may be derived as follows:

$$\dot{m} = \rho v \qquad (1)$$

$$\frac{d\dot{m}}{dp} = \rho \frac{dv}{dp} + v \frac{d\rho}{dp} \qquad (2)$$

Where: $\dot{m}$ = mass flow rate per unit area
ρ = gas density
v = velocity
p = pressure

But "Y", the acoustic admittance, is defined by: $Y = -\frac{dv}{dp}$

hence: $$Y = \frac{d\dot{m}}{dp} - v\frac{d\rho}{dp} \qquad (3)$$

If the period of the oscillation is long compared to the combustion time of the reactants, the combustion gas will be emitted from the flame zone with a temperature which is the same at all times, since the adiabatic flame temperature is essentially independent of pressure.

For a constant flame temperature,

$$\frac{dp}{d\rho} = RT = \frac{p}{\rho} \qquad (4)$$

leading to $$Y\frac{p}{v} = 1 - \frac{d\dot{m}/\dot{m}}{dp/p}$$

writing $\mu = \frac{d\dot{m}}{\dot{m}}$ and $\epsilon = \frac{dp}{p}$, we immediately arrive at

$$Y = -\frac{V}{p}\left(\frac{\mu}{\epsilon} - 1\right) \tag{5}$$

Equation 5 may be compared with:

$$Y = -\frac{V}{p}\left(\frac{\mu}{\epsilon} - \frac{1}{\gamma}\right) \tag{6}$$

as derived by McClure*.

The difference is that McClure took the Lagrangian pressure-density derivative while in this report the Eulerian derivative was used. It is believed that this approach is the correct one.

This comparison shows that the use of a constant flame temperature implies greater stability, although the presence of other gain and loss mechanisms means that stability or instability of a rocket chamber can't be explained in terms of the acoustic admittance of the propellant alone.

IV. ENTROPY WAVE HYPOTHESIS AND SUGGESTED EXPERIMENTS

In the derivation of equation (5) it was assumed that the combustion gas is emitted from the flame at essentially the same temperature throughout the pressure cycle. However, when the gas enters the duct, its temperature will oscillate in phase with pressure as illustrated in Figure 3. In this time-temperature plot it can be seen that the gas which is emitted from the flame at low pressure is compressed by subsequent pressure waves and raised to temperatures higher than the flame temperature. At the same time, gas emitted at low pressure is later cooled below the flame temperature by rarefactions. Accordingly, at any moment,

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* McClure assumes that a particle of gas traversing the flame is subject to an adiabatic compression, so that:

$$\frac{dp}{d\rho} = \frac{\gamma p}{\rho}$$

which may be substituted into equation 3 to give:

$$Y = \frac{d\dot{m}}{dp} - \frac{v\rho}{\gamma p}$$

Then, writing $Yp/v = 1/\gamma - d\dot{m}/\dot{m}/dp/p$ and $\epsilon = dp/p$, leads to equation 6.

there should exist in the duct waves of temperature which should be observable by their luminosity. An idealized representation of the appearance of the gases in the duct is shown in Figure 4.

An alternate description of the condition of the gases in the duct may be made through the introduction of the entropy wave concept.

For a polytropic gas:

$$S = S^{o} + C_{p} \ln T/T^{o} - R \ln P/P^{o} \qquad (7)$$

where the symbols have their usual meanings. If it is assumed that the particles are all emitted at the same flame temperature, then at emission:

$$S_{e} = \text{Constant}' - R \ln P_{e} \qquad (8)$$

but in an oscillating pressure field:

$$P_{e} = P_{avg} \; (1 + \epsilon \sin \omega t) \qquad (9)$$

substituting: $S_{e} = \text{Constant} - R \ln (1 + \epsilon \sin \omega t)$ (10)

Since $\ln (1 + x) = x$, for $x \ll 1$

$$Se = \text{Constant} - R \epsilon \sin \omega t$$

$$= R \epsilon \sin (\omega t - \pi) + \text{Constant} \qquad (11)$$

Thus, by this simple relationship, the pressure and entropy are expected to be 180° out of phase. In other words, the high entropy waves are emitted at pressure minima. The foregoing derivation assumed that time of combustion of a given particle was short compared to the time of oscillation. If the frequency of oscillation is very high, this simple derivation does not apply. The more complete derivation is being prepared for publication as a journal article.

Experiments are now being planned to test for the occurrence of entropy waves using the previously described oscillator (2). The use of back-lighting should help to resolve the possibility of "smoke", since the transmission of light through the combustion products would vary with "smoke" density.

An even more significant experiment may be employed to test the entropy wave hypothesis. A schematic of the equipment is shown in Figure 5. The experiment is effectively a time-resolved sodium "D" line reversal method for determining gas temperature, and is similar to a method used to study the temperature history behind a shock wave (3). For this purpose, small amounts of a sodium salt could be incorporated in the propellant. The source

temperature would be varied to obtain an indication of the gas temperature. The choice of a source will be dictated by the flame temperature of the propellant under study. A consideration of the entropy wave concept and adiabatic compression indicates that a 20% temperature variation is to be expected. If the initial results prove promising, other experiments might be conducted. Among these are first, a variation of the source temperature during a run in order to obtain a better measurement of the absolute temperature magnitude. A second technique involves the use of interference filters through which the gas luminosity could be photographed so that the temperature variations would be accentuated by the intensity of the sodium emission recorded on the film.

The pressure relationship can be obtained directly by the use of a gauge which can be mounted in the test section in the same plane as the three view windows.

V. PARTICLE TRACK METHOD

Observations of velocity profiles by means of luminous or illuminated particles have been used by some experimentalists to trace temperature profiles in laminar flames (4). The shapes of the flow pattern around various wings in low-speed wind tunnels have been studied with luminous tracers ejected into the gas stream(5). This type of technique has been considered as a possible method of determining the response function of the burning propellant surface.

The use of the track method consists of determining the velocity of the tracer as a function of time and position by either: (a) following individual particles through a number of frames from a movie film and determining velocity from the displacement-time history, (b) illuminating the flow field stroboscopically and determining the different velocities from the length of the lines which result on the film, or (c) following the progress of the tracer by a continuous writing streak or strip camera, and determining velocities from measurements of the slope of the curve. A knowledge of the velocity and pressure as a function of time gives either the admittance directly or, together with the continuity of mass equation, gives a knowledge of the mass flow rate as a function of time. Hence, the reaction of the propellant to pressure fluctuations may be readily determined.

The feasibility of using the particle track method will depend upon the correctness of results to be expected. To investigate the fidelity with which the particles follow the gas velocity, consider the classic Stokes equation for the equation of motion of a spherical particle in a gas flowing with an imposed sinusoidal velocity variation.

$$\ddot{x} = \alpha(\bar{V} + \tilde{V}\sin\omega t - \dot{x}) \qquad (12)$$

where $\bar{V}$ = average gas velocity
$\tilde{V}$ = amplitude of gas velocity variation
α = $3\pi\mu D/m = 18\mu/\rho D^2$
μ = gas viscosity
D = particle diameter
ρ = particle density

The solution of the differential equation (12) may be written:

$$\dot{x} = \bar{V}(1-e^{-\alpha t}) + \frac{\tilde{V}}{\sqrt{1+\frac{\omega^2}{\alpha^2}}} \cos(\omega t-\phi) + \frac{\alpha\tilde{V}\omega e^{-\alpha t}}{\alpha^2+\omega^2} \qquad (13)$$

where: $\phi = \tan^{-1}\left(\frac{\alpha}{\omega}\right)$

The last term in equation (13) is small compared with the first two terms since the magnitude of α is such that the exponential decays very rapidly and the pre-exponential term is less than unity.

Equation (13) indicates that three factors will affect the accuracy with which observed particle tracks will reflect the true gas velocity history. In the first term of equation (13), the expression ($1-e^{-\alpha t}$) represents the extent to which the particle attains the steady gas velocity. In the second term, the expression ($1 + \omega^2/\alpha^2$)$^{-1/2}$ represents the accuracy with which the particle track will indicate the amplitude of the gas velocity oscillations, while ϕ represents the phase lag between the gas velocity oscillation and the particle velocity oscillation. The effects of these factors may be considered separately.

To study the error from the first term, the time and distance required for a particle to reach 95% of the gas velocity were calculated, using a velocity of 500 cm/sec., ρ = 2.8, and μ = .063 centipoise. The results are as follows:

Particle Diameter (microns)	τ (msecs.)	x (cm.)
5	0.24	0.08
10	0.95	0.32
20	3.79	1.30

It should be mentioned, at this point, that the distances resulting are large compared to the combustion zone thickness (.01cm), thereby justifying the use of a steady flow, rather than a flow which initially undergoes a very steep acceleration to 500 cm/sec. in .01cm. This assumption greatly simplifies the equations and resulting calculation.

To study the error from the second term, calculations were made to determine the magnitude of the phase lag as well as the amplitude of $\dot{u}$ compared to $\tilde{V}$. The results are as follows:

D (microns)	Phase Lag		Relative Amplitude	
	100 cps	500cps	100 cps	500cps
5	2.8°	13.9°	1.00	.98
10	11.2°	44.7°	.98	.71
20	38.4°	75.9°	.78	.45

If we content ourselves with a 90% relative amplitude, then the criterion that $\omega/\alpha < 0.5$ can be imposed. This criterion leads to a 26° maximum limit in phase error, and also leads to the particle attaining 95% of steady gas flow velocity in less than ¼ period. Substituting, the criterion leads to the requirement that:

$$\rho D^2 f < 8.8 \times 10^4 \qquad \text{(D in microns)}$$

The results are plotted in Figure 7. The effect of the density of the particle should be noted. As a side note, the imposition of 95% relative velocity amplitude leads to $\omega/\alpha < 0.1$, giving a phase error of about 6°, and catching up times on the order of 1/20 period. This criterion drops the maximum frequency for a given particle by a factor of 5.

Consideration of the calculations reveals rapidly that very small particles must be made visible for the method to have any feasibility. As a result, good lighting and excellent optics will be required. First attempts will be made with the set-up shown in Figure 6. A flash bulb might be used because 25msecs. would give 10 periods at 500 cps. and allow 5 separate "families" of particles to be followed from the surface through the viewing section. The use of a chopper to give periodic illumination makes velocity determinations far simpler, but steady light could be used to give a complete history of a particle, even though absolute measurements are harder. The focusing can be varied depending on the light requirements. In the drawing, a "pencil" of light is used, so that fewer particles will be under observation, leading to greater clarity in interpretation. Focusing on a point as opposed to a line might give better results.

Experiments will initially be made using well-screened MgO, since this oxide has a melting point (3100° K) in excess of flame temperatures and should thus keep its shape well in the combustion gases.

VI. CONCLUDING REMARKS:

The general objectives of the experimental program during the next quarter are as follows:

1. The operating parameters of the oscillator driver will be investigated.

2. The combustion of propellants of various types under conditions of oscillating pressure will be studied by direct photographs.

3. Experiments to measure gas temperature and gas particle velocities will be instituted.

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2. R. H. W. Waesche, K. P. Hall, and M. Summerfield, "Research on Solid Propellant Instability", Initial Progress Report No. 564-a, 1 May 1961 to 31 July 1961, Princeton University Department of Aeronautical Engineering, August, 1961.

3. J. G. Clouston, A. G. Gaydon, and I. I. Glass, "Temperature Measurements of Shock Waves by the Spectrum Line Reversal Method," Proc. Roy. Soc. 248, 429 (1958).

4. R. M. Fristrom, R. Prescott, R. K. Neumann, and W. H. Avery, "Temperature Profiles in Propane-Air Flame Fronts, Fourth Symposium on Combustion," Williams and Wilkens, Baltimore. 1953, p. 267.

5. J. M. Bourot, "Chronophotographie des Champs Aerodynamiques" Scientific and Technical Publications of Air Ministry of France, No. 226, 1949.

TABLE I

Experimental conditions under which film strip was exposed:

Mean chamber pressure	- 500 psi
Pressure oscillation amplitude	± 125 psi
Mean burning rate	- 0.5 in./sec.
Frequency of oscillation	- 500 cps.
Camera speed	- 5000 frames/sec.
Propellant	- composite double-base, containing 2% Mg (through 400 mesh)
Theoretical flame temperature	- 3100° K

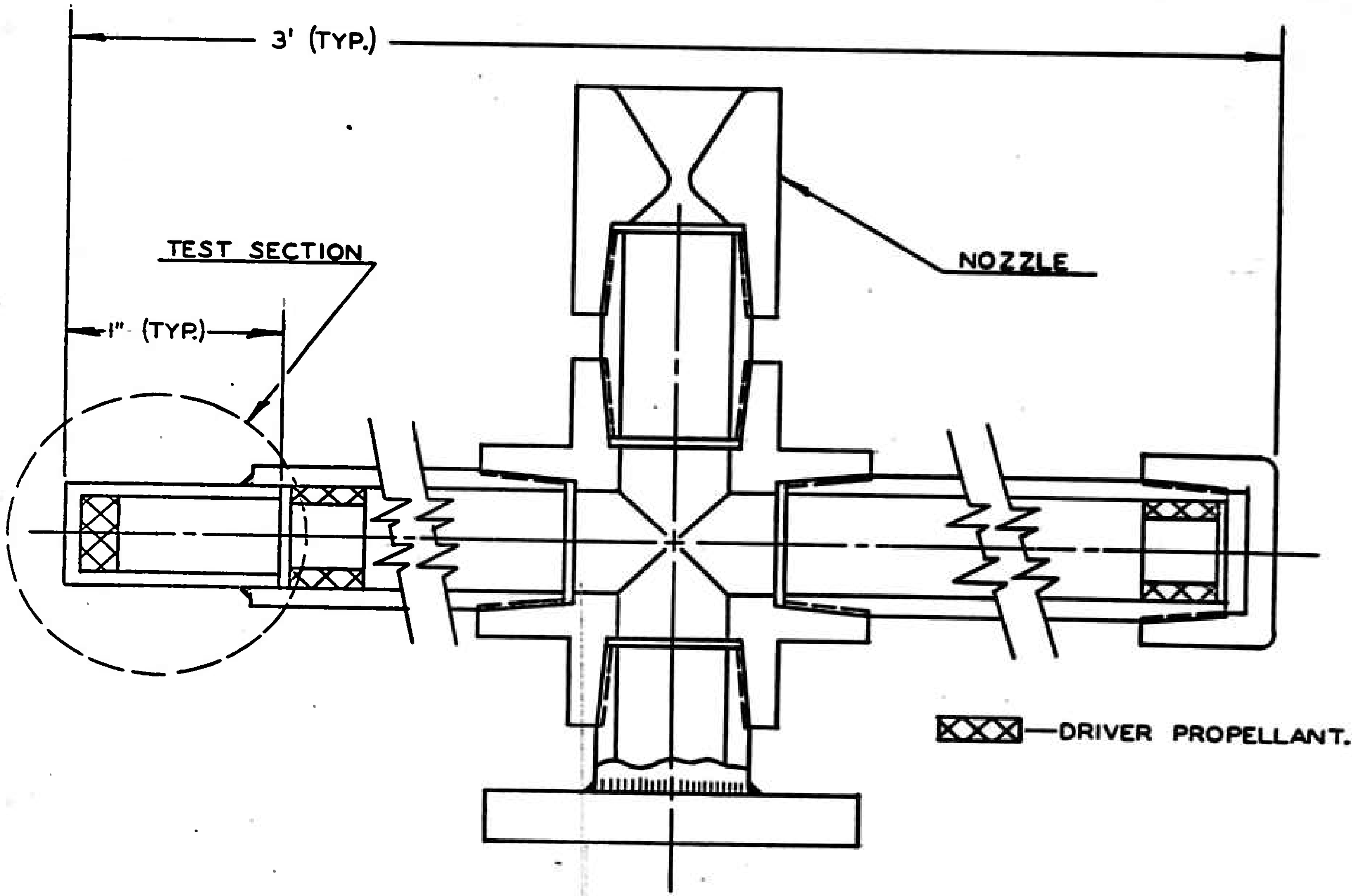

FIGURE 1

Experimental apparatus used at Rohm & Haas to photograph combustion of metallized propellant in an oscillating pressure field.

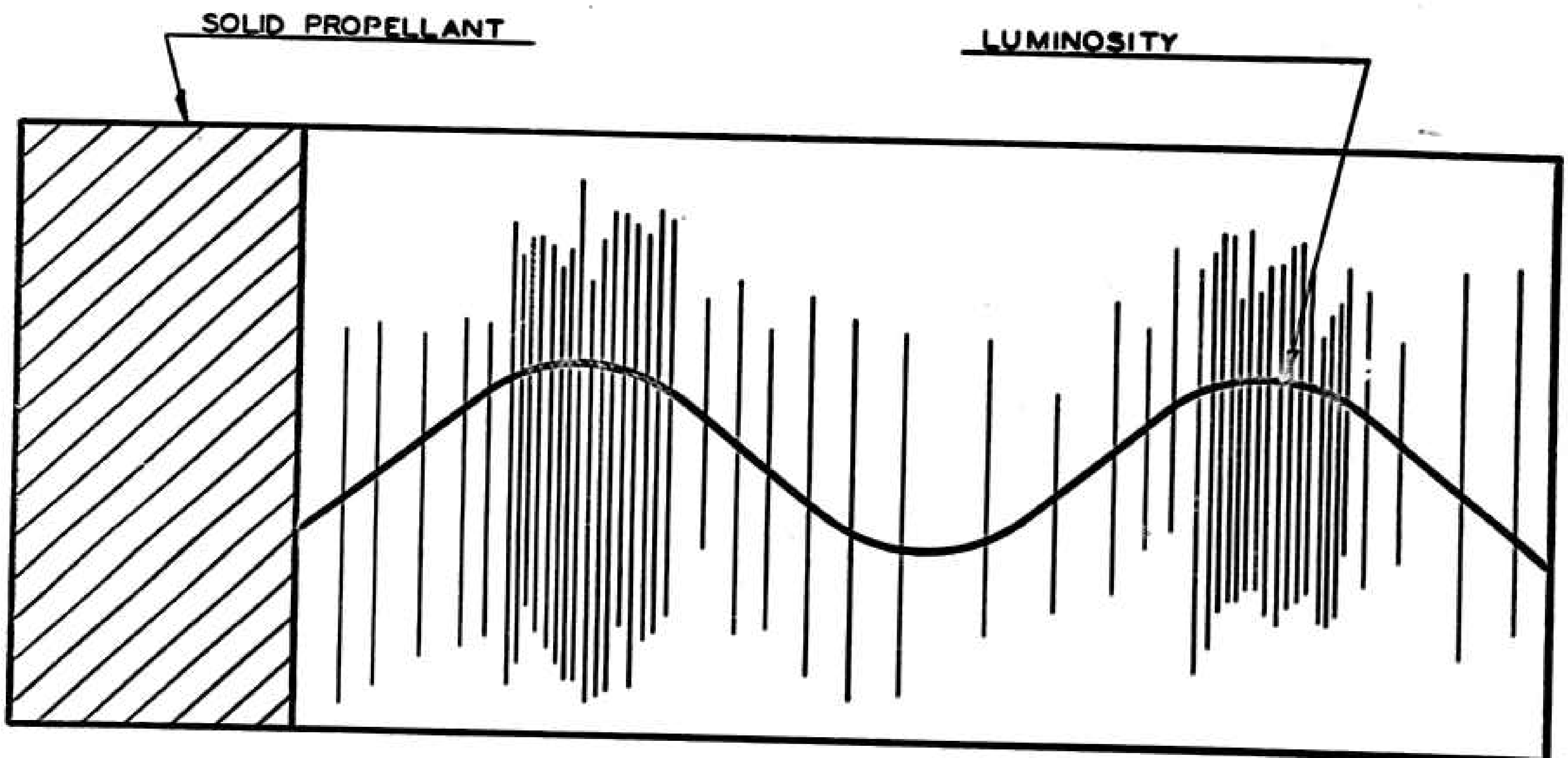

FIGURE 2

This is a representation of the appearance of a typical frame of the film strip which was analyzed.

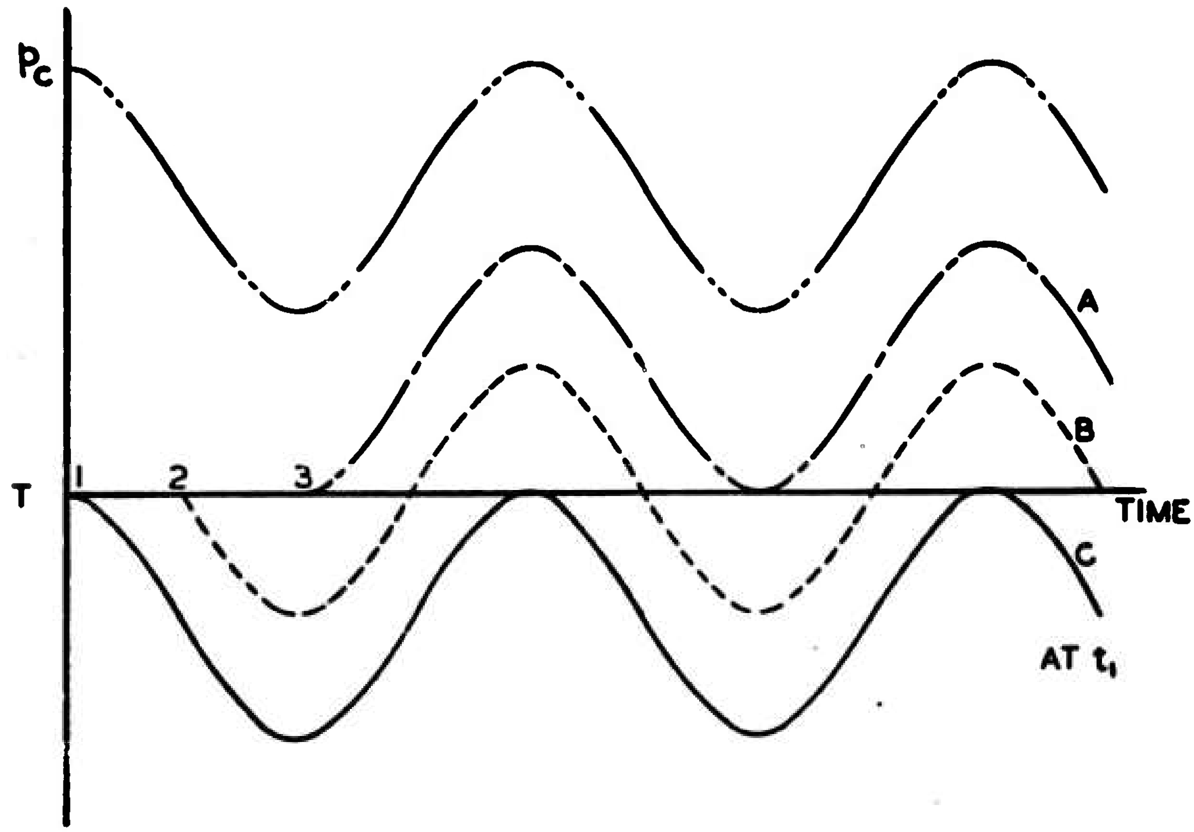

FIGURE 3

In this figure the pressure and temperature of gas packets emitted from the flame are plotted as a function of time. Particles emitted at various times during the pressure cycle all have the same temperature. However, when they enter the duct their different entropies are manifested as temperature differences. Thus, gas particle 1, emitted at high pressure and low entropy, senses an expansion and cooling to some lower temperature, and its temperature oscillates between this temperature and the flame temperature. Particle 3, emitted at low pressure and thus at high entropy, is subsequently compressed and heated to some higher temperature. The temperature of this particle emitted at low pressure oscillates between this higher temperature and the flame temperature.

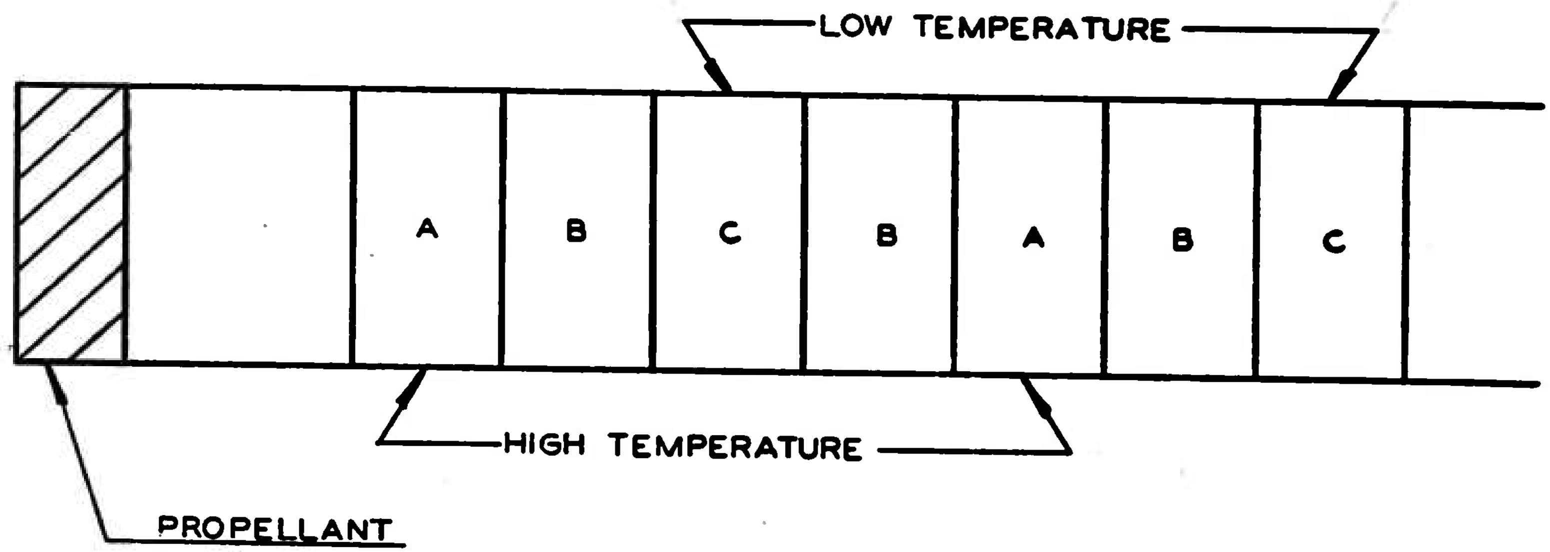

FIGURE 4

This is an idealized representation of the appearance of the gases in the duct at some time during the combustion of a solid propellant in an oscillating pressure environment. The regions "A" are at high temperature and these regions can be distinguished by their high luminosity. At the same time, regions labelled "C" are cooler and thus of lower luminosity.

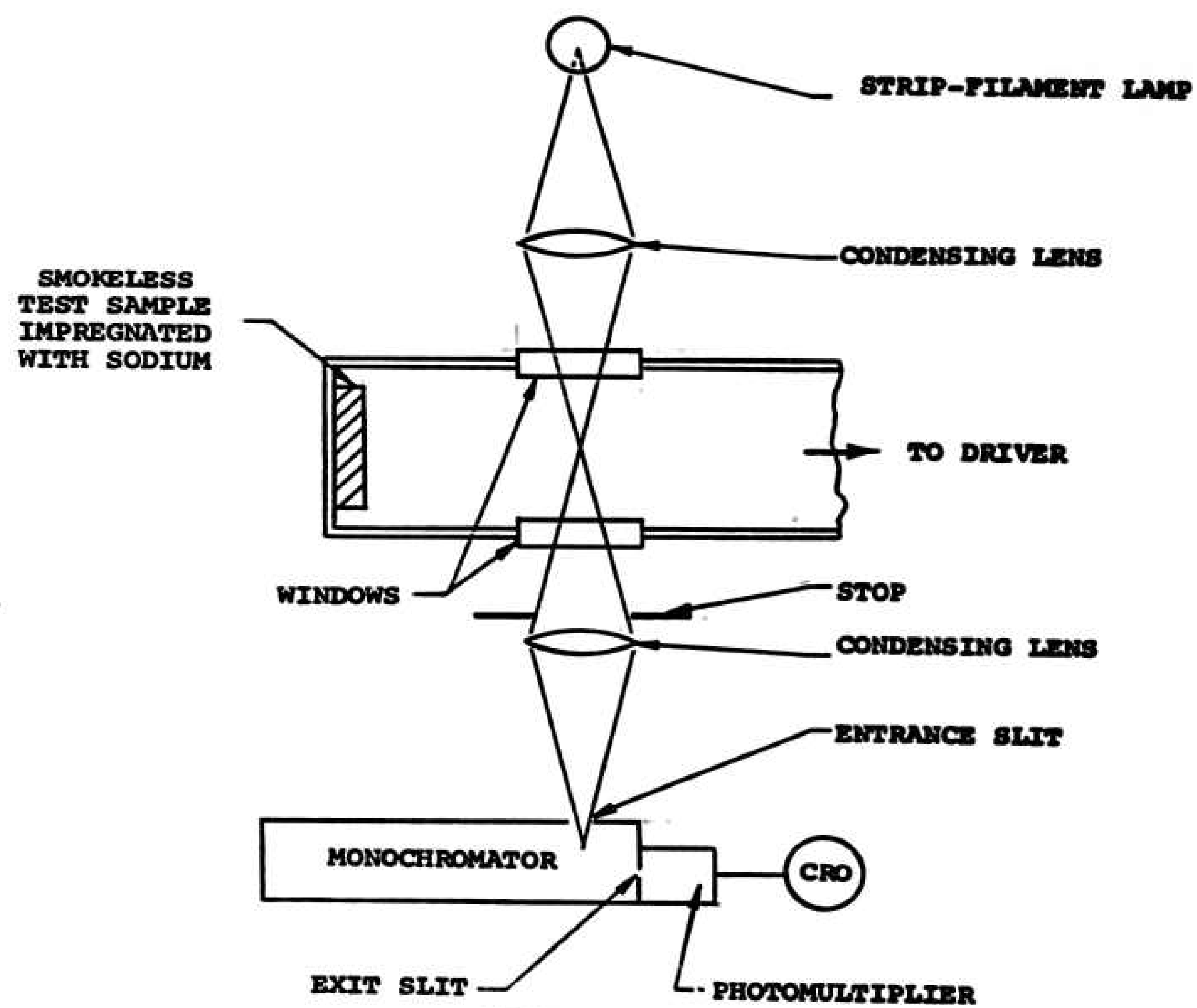

FIGURE 5

This figure is a schematic representation of an experimental apparatus designed to make a time resolved measurement of the temperature of the luminous gases emitted from a burning solid propellant surface.

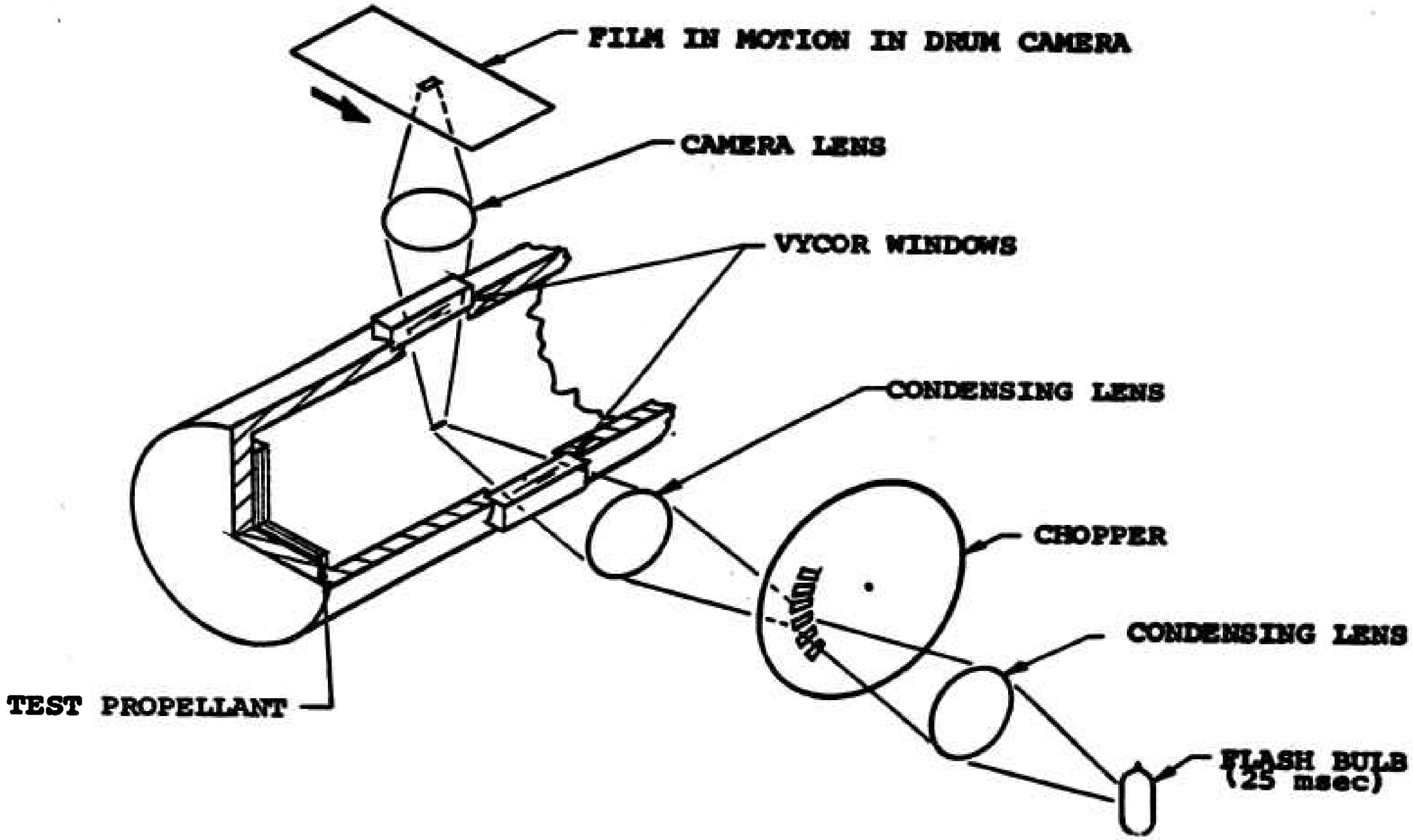

FIGURE 6

This figure is a schematic representation of an experimental apparatus designed to observe particles moving under the influence of an oscillating pressure field. The chopped radiation of a known frequency will give streaks on the film whose lengths will be proportional to the particle velocity.

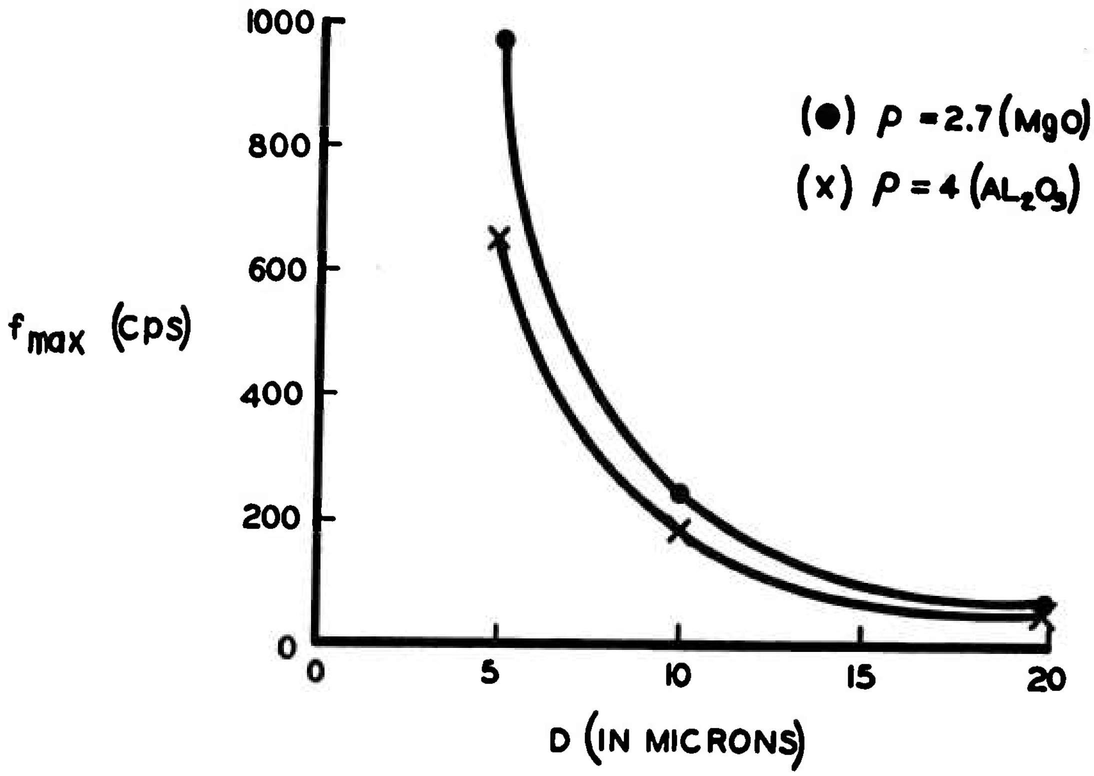

FIGURE 7

In this figure are plotted the maximum sizes of particles which will faithfully (within 90% of true amplitude) follow a velocity oscillation of a given frequency.

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